B.Sc. part - II (Sub) Diff. Calculus Date 09-07-2020.

Problems based on Derivatives of Higher order

Q.1. If $p = a^2\cos^2\theta + b^2\sin^2\theta$, Show that

$$p + \frac{d^2p}{d\theta^2} = 2a^2 + 2b^2 - 3p$$

Soln: We have, $p = a^2\cos^2\theta + b^2\sin^2\theta$ —— (1)

Differentiating both sides w.r.t. θ, we get

$$\frac{dp}{d\theta} = -2a^2\cos\theta\sin\theta + 2b^2\sin\theta\cos\theta$$

$$= -a^2\sin2\theta + b^2\sin2\theta$$

$$= (b^2 - a^2)\sin2\theta \quad \text{—— (2)}$$

Again, differentiating both sides w.r.t. θ, we get

$$\frac{d^2p}{d\theta^2} = 2(b^2 - a^2)\cos2\theta \quad \text{—— (3)}$$

Adding (1) and (3), we get

$$p + \frac{d^2p}{d\theta^2} = 2(b^2 - a^2)(\cos^2\theta - \sin^2\theta) + a^2\cos^2\theta + b^2\sin^2\theta$$

$$= \cos^2\theta(2b^2 - 2a^2 + a^2) + \sin^2\theta(2a^2 - 2b^2 + b^2)$$

$$= \cos^2\theta(2b^2 - 3a^2 + 2a^2) + \sin^2\theta(2a^2 - 3b^2 + 2b^2)$$

$$= \cos^2\theta(2b^2 - 3a^2 + 2a^2) + \sin^2\theta(2a^2 - 3b^2 + 2b^2)$$

$$= (2a^2 + 2b^2)(\cos^2\theta + \sin^2\theta) - 3(a^2\cos^2\theta + b^2\sin^2\theta)$$

$$= (2a^2 + 2b^2) - 3p$$

$$\Rightarrow p + \frac{d^2p}{d\theta^2} = 2a^2 + 2b^2 - 3p$$

Q.2. If $\sqrt{x+y} + \sqrt{y-x} = c$, Show that

$$y_2 = \frac{2}{c^2}$$

Soln. We have, $\sqrt{x+y} + \sqrt{y-x} = c$

Squaring both sides, we get

$$(\sqrt{x+y})^2 + (\sqrt{y-x})^2 + 2\sqrt{y^2-x^2} = c^2$$

$$\Rightarrow x + y + y - x + 2\sqrt{y^2-x^2} = c^2$$

$$\Rightarrow 2\sqrt{y^2-x^2} = c^2 - 2y$$

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Rest part of Q. 2.

Again, squaring both sides, we get

$$\left(2\sqrt{y^2-x^2}\right)^2 = (c^2-2y)^2$$

$$\Rightarrow 4(y^2-x^2) = c^4 - 4c^2y + 4y^2$$

$$\Rightarrow 4x^2 - 4c^2y + c^4 = 0$$

Diff. both sides w.r.t. x, we get

$$8x - 4c^2y_1 = 0$$

Again, diff. both sides, w.r.t. x, we get

$$2 - c^2y_2 = 0 \Rightarrow y_2 = \frac{2}{c^2}$$ Proved.

Q. 3. If $ax^2+2hxy+by^2=1$, Show that

$$y_2 = \frac{h^2-ab}{(hx+by)^3}$$

Soln: We have, $ax^2+2hxy+by^2=1$ ——— (1)

Diff. w.r.t. x, we get

$$2ax + 2hy + 2hxy_1 + 2byy_1 = 0$$

$$\Rightarrow y_1(hx+by) = -(ax+hy)$$

$$\Rightarrow y_1 = -\frac{ax+hy}{hx+by}$$ ——— (2)

Diff. (2) w.r.t. x, we get

$$y_2 = -\frac{(a+hy_1)(hx+by)-(ax+hy)(h+by_1)}{(hx+by)^2}$$

$$= -\frac{ahx+aby+h^2xy_1+bhyy_1-ahx-abxy_1-h^2y-bhyy_1}{(hx+by)^2}$$

$$= -\frac{xy_1(h^2-ab)-y(h^2-ab)}{(hx+by)^2} = \frac{h^2-ab}{(hx+by)^2}(y-xy_1)$$

$$= \frac{h^2-ab}{(hx+by)^2}\left(y+x\frac{ax+hy}{hx+by}\right) \quad [\text{by } (2)$$

$$= \frac{h^2-ab}{(hx+by)^2}\left(\frac{hxy+by^2+ax^2+hxy}{hx+by}\right)$$

$$= \frac{h^2-ab}{(hx+by)^3}(ax^2+2hxy+by^2)$$

Hence, $y_2 = \frac{h^2-ab}{(hx+by)^3}\cdot 1 = \frac{h^2-ab}{(hx+by)^3}$ [by (1)]

Proved.